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DETERMINATION OF THE MAXIMAL VALUES OF THE GRAVITATIONAL DRIFT AND THE DRIFT DUE TO IRREGULAR RIGIDITY IN INTEGRATING FLOATED-TYPE GYROSCOPES *74

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Derivation of formulas for maximum values of drift in integrating floated-type gyroscopes, proceeding from expressions for momenta due to the imbalance and irregular rigidity of the gyro-unit and assuming that the float is absolutely rigid while the gyromotor structure is not evenly rigid and has a fixed axis. The results are discussed. author

The moment of noise acting on the gyro unit of an integrating floated-type gyroscope in the general case is composed of moments not dependent upon acceleration and of moments dependent upon acceleration. The first include moments produced by flexible current conductors and convection currents of a fluid, the reactive moments of the angle and momentum sensors, etc. The moments dependent upon acceleration are the moments produced by the instability and irregular rigidity of the gyro unit. The occurrence of gyroscope drift from the irregular rigidity of the suspension system was first indicated by A.Yu.Ishlinskiy (Bibl.1).

When the instrument moves with linear acceleration $\bar{a}$, the lift force and the weight (more exactly, the "apparent weight") of the gyro unit (Bibl.2) will be respectively equal to m_1w and mw ($m_1 = \rho V$ is the mass of the liquid in the volume V of the gyro unit, m is the mass of the gyro unit, $w = |\bar{g} - \bar{a}|$, and g is the acceleration of gravity), and the floating axis of the gyro unit is di-

* Numbers in the margin indicate pagination in the original foreign text.

rected along the "apparent vertical". Therefore, at $a \neq 0$ the difference between the lift force and the force of the weight of the gyro changes by $|\bar{g} - \bar{a}|/g$ times in comparison with its value at $a = 0$; in this case the sign of the difference remains unchanged. As a consequence of this, the moment due to instability of the gyro will be proportional to the first power of the projection of the vector $\bar{w} = \bar{g} - \bar{a}$ onto the transverse plane of the instrument yz_0 (y and z_0 are the output and lateral axes of the instrument). Correspondingly, the drift caused by this moment will also be proportional to the first power of the projection of the vector $\bar{w}$ onto the plane yz_0.

Since the drift due to the gyro unit instability is a function of acceleration, it can be estimated in $\frac{\text{deg/hr}}{g}$, i.e., by the value of the angular drift velocity observed on the ground in the absence of translational velocities ($a = 0$). The drift caused by instability of the gyro observed on the ground in the absence of translational velocities ($a = 0$) and with a horizontal position of the output axis of the instrument x will be called the gravitational /75 drift. At $a \neq 0$ and with a horizontal position of the x-axis, the drift due to instability of the gyro will be equal to the gravitational drift multiplied by the value of the projection of the vector $\bar{w} = \bar{g} - \bar{a}$ onto the yz_0 plane, expressed in fractions of g.

For floating gyroscopes, the moment caused by the irregular rigidity of the gyro unit is a consequence of the fact that, for an unequally rigid gyro unit, the elastic displacements of the center of gravity and of the center of pressure occur in directions that do not coincide with the direction of the action of the forces causing these displacements. This moment, and thus also the resultant drift, are proportional to the second power of the projection of the vector $\bar{w} = \bar{g} - \bar{a}$ onto the transverse plane of the instrument yz_0. In conformity with this, the drift caused by irregular rigidity of the gyro unit should be estimated in $\frac{\text{deg/hr}}{g^2}$, i.e., it can be estimated by the value of the angular drift velocity observed on the ground in the absence of translational velocities ($a = 0$).

Originally, the gyro unit of an integrating floated-type gyroscope consisted of a frame with a gyro motor and a hollow cylinder which was placed on the side parts of the frame, made in the form of disks, and hermetically connected with them. However, in the course of time, the frame was eliminated to impart greater rigidity to the gyro unit, and its functions were taken over by the float. The float consisted of two parts (each in the form of a container) hermetically connected with each other. A gyro motor was preliminarily mounted to one of them. With this design of the gyro unit it was possible to make the gyro motor equally rigid and the float absolutely rigid, for all practical purposes, relative to the maximal possible values of the forces acting on it.

Assuming that the float for practical purposes is absolutely rigid but the design of the gyro motor does not satisfy the condition of equal rigidity and is made with a fixed axle, we will derive the expressions for the moments M_2 and M_3 caused, respectively, by instability and irregular rigidity of the gyro unit. For this, we will introduce the following symbols (see diagram):

Point O = track of the output axis of the instrument x (axis of rotation of the gyro unit);

z = axis of rotation of the rotor of the gyro motor;

y = axis perpendicular to the x- and z-axes;

η and ζ = horizontal and vertical axes;

θ = angle of inclination of the z-axis to the plane of the horizon;

g = acceleration of gravity;

m_1 = mass of fluid in the volume of the gyro unit;

r_1; ψ_1 = polar coordinates of the center of pressure O_1;

m_2 = mass of gyro unit without gyro motor;

r_2; ψ_2 = polar coordinates of the point O_2 of application of a force of weight m_2g;

m_3 = mass of gyro motor; /76

r_3; ψ_3 = polar coordinates of the point O_3 at which the center of mass of the gyro motor would be located if the gyro motor were absolutely rigid;

m_4 = mass of a rotor component of the gyro motor, subject to elastic displacement parallel to the y-axis and parallel to the z-axis;

$m_3 - m_4$ = mass of an axle component of the gyro motor with stator, subject to elastic displacement only parallel to the y-axis;

O_4 = point of application of a force of weight $(m_3 - m_4)g$;

O_5 = point of application of a force of weight m_4g;

$m = m_2 + m_3$ = mass of entire gyro unit;

r, ψ = polar coordinates of the center of mass of the entire gyro unit with an absolutely rigid gyro motor;

K_{3y} = transverse rigidity of an axle component of the motor with stator;

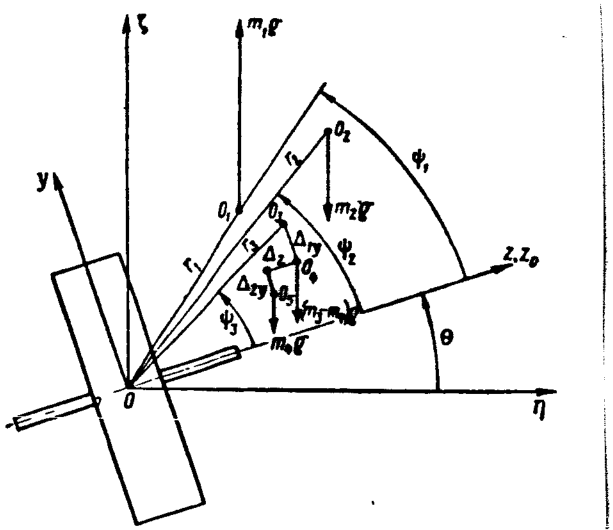
K_{4y} and K_{4z} = radial and axial rigidities of a component of the rotor with bearings;

$$\Delta_{1y} = \frac{gm_3}{K_{3y}} \cos\theta \tag{1}$$

= elastic displacement of the center of mass of an axle component, with the stator parallel to the y-axis;

$$\Delta_{2y} = \frac{gm_4}{K_{4y}} \cos\theta; \quad \Delta_z = \frac{gm_4}{K_{4z}} \sin\theta \tag{2}$$

= elastic displacements of the center of mass of a rotor component with bearings relative to the center of mass of an axle component with the stator parallel to the y- and z-axes, respectively. /77



Derivation of the Expression for the Moment $M_{2,3}$ Caused by Instability and Irregular Rigidity of the Gyro Unit

On the basis of eqs.(1) and (2) it can be derived that the forces, depicted in the diagram, produce the following moment about the x-axis:

$$M_{2.3} = gmr\cos(\theta+\psi) - gm_1r_1\cos(\theta+\psi_1) + \frac{g^2}{2}\left[\frac{m_3^2}{K_{3y}} + m_4^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right)\right]\sin 2\theta, \tag{3}$$

where

$$r = \frac{1}{m}\sqrt{(m_2r_2)^2 + (m_3r_3)^2 + 2m_2m_3r_2r_3\cos(\psi_2 - \psi_3)},$$

$$\tan\psi = \frac{m_2r_2\sin\psi_2 + m_3r_3\sin\psi_3}{m_2r_2\cos\psi_2 + m_3r_3\cos\psi_3}.$$

In eq.(3), the first two terms represent the moment M_2 caused by the instability of the gyro unit relative to the x-axis. Thus,

$$M_2 = g\,[mr\cos(\theta + \psi) - m_1 r_1 \cos(\theta + \psi_1)]. \qquad (4)$$

The third term is the moment M_3 caused by the irregular rigidity of the gyro motor. Consequently,

$$M_3 = \frac{[illegible]}{2}\left[\frac{[illegible]}{K_{3y}} [illegible] \left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right)\right]\sin 2\theta. \qquad (5)$$

The maximum value of the moment M_2

$$M_{2\,max} = g\sqrt{(mr)^2 + (m_1 r_1)^2 - 2mm_1 rr_1 \cos(\psi_1 - \psi)} \qquad (6)$$

is obtained at an angle θ equal to

$$\theta^* = \tan^{-1}\frac{m_1 r_1 \sin\psi_1 - mr\sin\psi}{mr\cos\psi - m_1 r_1 \cos\psi_1}. \qquad (7)$$

Using the equalities (6) and (7), we can present eq.(4) in the form of

$$M_2 = M_{2\,max}\cos(\theta - \theta^*). \qquad (8)$$

The moment M_2, in principle, can be equated to zero for any value of the angle θ, by various methods. However, so that $M_2 = 0$ at any possible value of the mass m_1, which is a function of the temperature of the fluid, it is necessary that $r = r_1 = 0$, i.e., that the center of mass and the center of pressure of the gyro unit be on its axis of rotation. With any other method of making the moment M_2 vanish, the equality $M_2 = 0$ will not be invariant with respect /78 to the possible values of the mass m_1. For example, if at a certain temperature of the fluid the moment M_2 is made to vanish by reducing the center of mass and the center of pressure to a single point which does not lie on the axis of rotation of the gyro unit ($m = m_1$, $\psi = \psi_1$, $r = r_1 \neq 0$), then a change in temperature will cause the equality $m_1 = m$ to be disturbed owing to a change in m_1, as a consequence of which, at $r = r_1 \neq 0$, the moment M_2 will cease being equal to zero.

It is quite difficult to make the moment M_2 precisely and stably equal to zero. A decrease in the moment M_2 to a value close to zero is achieved by careful balancing of the gyro unit by a special procedure with the use of high-precision devices.

It is apparent from eq.(5) that, on any change in the angle θ, the moment M_3 will change with a doubled frequency and its peak amplitude will be

$$M_{3\,\max} = \frac{g^2}{2}\left[\frac{m_3^2}{K_{3y}} + m_4^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right)\right] \tag{9}$$

yielding, for the angles

$$\theta = \frac{\pi}{4}(2k+1);\ \ k = 0,\ 1,\ 2,\ \ldots \tag{10}$$

So that the moment M_3 will be equal to zero at any value of the angle θ, it is necessary to observe the following condition:

$$\frac{m_3^2}{K_{3y}} + m_4^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right) = 0,$$

which is the condition of equal rigidity in the case under consideration. When exact satisfaction of this condition is difficult, the residual value of the moment caused by the irregular rigidity can be compensated by a special compensator, which is a small mass mounted to a flat spring. By rotating the spring about its long axis, we can change the sense of its deformation and thus also the direction of displacement of the mass. By experimentally selecting the position of the spring, we can theoretically achieve complete equal rigidity of the system. This compensator, as it were, reduces the rigidity of the gyro unit in the direction of the axis along which it has a greater rigidity.

In the general case, drift of an integrating floated-type gyroscope is caused by the moment acting about the x-axis

$$M = M_1 + M_2 + M_3,$$

where M_1 is the moment of noise which is not dependent upon acceleration, in other words, the moment due to causes not associated with instability and unequal rigidity of the gyro unit. The moments M_2 and M_3 are determined by eqs.(8) and (5). Correspondingly, in the general case the angular drift velocity is /79

$$\omega_d = \omega_{d1} + \omega_{d2} + \omega_{d3}, \tag{11}$$

where

$$\omega_{d1} = \frac{M_1}{H}, \quad \omega_{d2} = \frac{M_2}{H}, \quad \omega_{d3} = \frac{M_3}{H}. \tag{12}$$

Here, H is the intrinsic (kinetic) moment of the gyroscope.

We will designate

$$\omega_{d1\,max} = \frac{M_{1\,max}}{H} \tag{13}$$

as the maximum value of the velocity of drift caused by the moment of noise which is independent of acceleration;

$$\omega_{d2\,max} = \frac{M_{2\,max}}{H} \tag{14}$$

as the maximum value of the velocity of drift caused by instability of the gyro unit;

$$\omega_{d3\,max} = \frac{M_{3\,max}}{H} \tag{15}$$

as the maximum value of the velocity of drift caused by irregular rigidity of the gyro unit (in our case, of the gyro motor).

When designing an integrating floated-type gyroscope, the kinetic moment H and the figure of merit D of the gyro motor ($D = H/P_3$, where P_3 is the weight of the gyro motor) should be selected as a function of the given values of ω_{d2max} and ω_{d3max} according to the following formulas derived from eqs.(14), (6), (15), and (9):

$$\frac{ar}{D}(1 + bc) = \omega_{d2\,max}; \tag{16}$$

$$\frac{H}{D^2}\left[\frac{1}{K_{3y}} + d^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right)\right] = \omega_{d3\,max}, \tag{17}$$

where

$a = P/P_3$ (P is the weight of the gyro unit);

$b = Q/P$ (Q is the weight of the fluid in the volume of the gyro unit);

$c = r_1/r$;

$d = P_4/P_3$ (P_4 is the weight of the rotor of the gyro motor). The other designations are as before.

Equation (16) is derived on the assumption that $\psi_1 - \psi = \pi$, since in this case the moment M_{2max} determined by eq.(6) has, other conditions being equal, the highest value.

So that the component of the angular drift velocity, which does not /80 depend on the acceleration, will not exceed the maximal permissible value ω_{d1max}, the following inequality must be observed

$$M_{1\,max} \leqslant H\omega_{d1\,max}. \tag{18}$$

With an equally rigid gyro motor, H and D should be determined by eqs.(18) and (16).

As is known, the angular drift velocity of integrating floated-type gyroscopes is determined by a special dynamic stand which permits measuring it both in a horizontal position of the x-axis and at various values of the angle θ and in the case where the x-axis is vertical. Let us assume that the angular drift velocity measured at angles θ equal to 0, $\frac{\pi}{4}$, and $\frac{\pi}{2}$, is respectively equal to $\omega_d(0)$, $\omega_d\left(\frac{\pi}{4}\right)$, and $\omega_d\left(\frac{\pi}{2}\right)$. The angular drift velocity measured at a vertical position of the x-axis will be denoted by $\omega_{d\,vert}$. This velocity can be taken for the velocity ω_{d1} since, at a vertical position of the x-axis, $M_2 = M_3 = 0$ so that the drift is caused only by the moment M_1.

We will show how to determine ω_{d2max} and ω_{d3max} if $\omega_{d\,vert}$, $\omega_d(0)$, $\omega_d\left(\frac{\pi}{4}\right)$, and $\omega_d\left(\frac{\pi}{2}\right)$ are known.

From eqs.(4) and (6) we see that

$$M_{2\,max} = \sqrt{M_2^2(0) + M_2^2\left(\frac{\pi}{2}\right)}, \tag{19}$$

where $M_2(0)$ and $M_2\left(\frac{\pi}{2}\right)$ are the values of the moment M_2, respectively, for $\theta = 0$ and $\theta = \frac{\pi}{2}$.

Substituting eq.(19) into eq.(14), we obtain

$$\omega_{d2\,max} = \sqrt{\omega_{d2}^2(0) + \omega_{d2}^2\left(\frac{\pi}{2}\right)}, \tag{20}$$

where $\omega_{d2}(0)$ and $\omega_{d2}\left(\frac{\pi}{2}\right)$ are the angular velocities of gravitational drift for $\theta = 0$ and $\theta = \frac{\pi}{2}$, respectively. From eq.(5) and from eq.(12) for ω_{d3} we see that, for these values of the angle θ, the angular drift velocity is $\omega_{d3} = 0$; consequently, from eq.(11) it follows that

$$\left.\begin{aligned} \omega_{d2}(0) &= \omega_d(0) - \omega_{d,\,vert.} \\ \omega_{d2}\left(\frac{\pi}{2}\right) &= \omega_d\left(\frac{\pi}{2}\right) - \omega_{d,\,vert.} \end{aligned}\right\} \tag{21}$$

Substituting eq.(21) into eq.(20), we derive the final formula for calculating the maximum value of the gravitational drift: /81

$$\omega_{d2\,max} = \sqrt{\left[\omega_d(0) - \omega_{d.\,vert.}\right]^2 + \left[\omega_d\left(\frac{\pi}{2}\right) - \omega_{vert.}\right]^2}. \tag{22}$$

We see from eq.(8) that ω_{d2max} will occur at angles θ equal to θ^* and $\pi + \theta^*$. At both angles, the direction of the drift will be the same.

From eqs.(4) and (7), we obtain

$$\theta^* = \tan^{-1}\frac{M_2\left(\frac{\pi}{2}\right)}{M_2(0)}.$$

Using the equality (12) for ω_{d2} and eq.(21), we get

$$\theta^* = \tan^{-1}\frac{\omega_{d2}\left(\frac{\pi}{2}\right) - \omega_{d,\,vert.}}{\omega_{d2}(0) - \omega_{d,\,\text{верт}}}. \tag{23}$$

Equations (5), (9), and (15) directly show that ω_{d3max} will occur at angles θ determined by the equality (10) and, in particular, when $\theta = \frac{\pi}{4}$. Therefore, on the basis of eq.(11) we can write

$$\omega_{d3\,max} = \omega_d\left(\frac{\pi}{4}\right) - \omega_{d2}\left(\frac{\pi}{4}\right) - \omega_{d,\,vert.} \tag{24}$$

where $\omega_{d2}\left(\frac{\pi}{4}\right)$ is the value of ω_{d2} at $\theta = \frac{\pi}{4}$.

Assuming $\theta = \frac{\pi}{4}$, in eq.(4), we obtain

$$M_2\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\left[M_2(0) + M_2\left(\frac{\pi}{2}\right)\right].$$

Substituting this value of $M_2\left(\frac{\pi}{4}\right)$ into eq.(12) for ω_{d2} we find that

$$\omega_{d2}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\left[\omega_{d2}(0) + \omega_{d2}\left(\frac{\pi}{2}\right)\right].$$

Substituting this expression and the equalities (21) into (24), we obtain the final formula for calculating the maximum value of drift caused by the irregular rigidity of the gyro unit: /82

$$\omega_{d3\,max} = \omega_d\left(\frac{\pi}{4}\right) - \frac{\sqrt{2}}{2}\left[\omega_d(0) + \omega_d\left(\frac{\pi}{2}\right)\right] + (\sqrt{2} - 1)\,\omega_{d,\,vert.} \tag{25}$$

Thus, for calculating the values of ω_{d2max}, ω_{d3max}, and θ^* it is necessary to know [as we see from eqs.(22), (25), and (23)] the angular drift velocities $\omega_d(0)$, $\omega_d\left(\frac{\pi}{4}\right)$, $\omega_d\left(\frac{\pi}{2}\right)$, and $\omega_{d\,vert}$. However, it should be noted that, with a horizontal position of the x-axis, the velocity ω_{d1} may differ somewhat from $\omega_{d\,vert}$. Furthermore, at various values of the angle θ, the velocity ω_{d1} can also have various values. Therefore, along with the indicated simple method of determining ω_{d2max} and ω_{d3max}, another method should be recommended which involves measuring ω_d in the horizontal position of the x-axis and various values of the angle θ from 0 to 360° at equal intervals, for example every 30°. Then, the graph of the dependence of ω_d on θ is analyzed by the harmonic balance method.

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